

Make a Tower to the Moon

With a sheet of paper and some mental maths, writes **Norman Hadley**

How many times must you fold a piece of paper to reach the moon, and can you work it out in your head? The question is based on an old chestnut that asks children how many times they can fold a piece of paper. After six folds, they're about done, and anyone who thinks they can get to seven is using either very thin paper or a liberal definition of the word 'fold'. At best, you can make a slight crease after the sixth fold.

Children instinctively believe the restriction arises from the paper size, and go off in search of a bigger sheet. However, even a sheet big enough to cover a king-size bed will only fold six times.

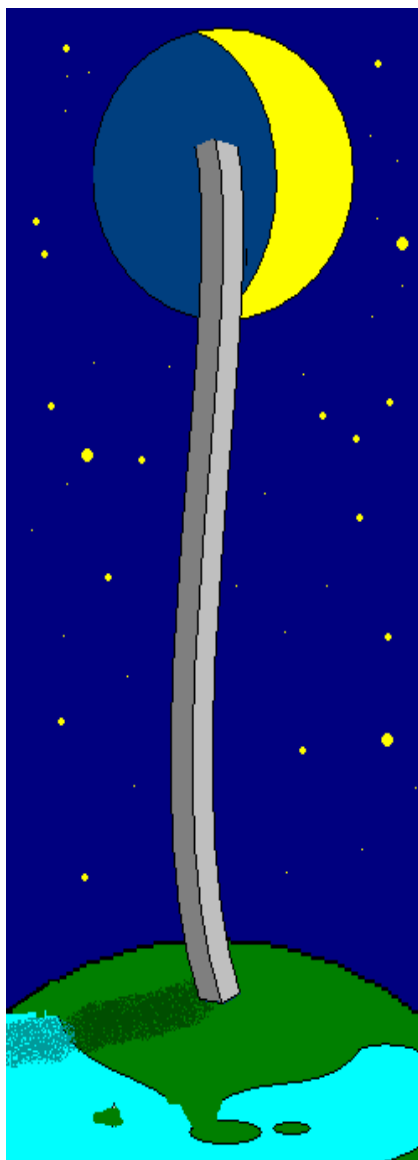
The restriction is not in the size but in the thickness. Each fold doubles the thickness until it reaches a level that will not fold any more.

The trouble is; we're nowhere near the moon! However, mathematicians have a neat trick when physical constraints get in the way of a good theorem: ignore them. So imagine we've been to W H Smith and bought a really big sheet of super-stretchy paper that will fold repeatedly without complaint. If it's the maths we're interested in, it's the concept of doubling every time that matters.

Children are intrigued by the concept of doubling because it generates big numbers surprisingly quickly. The series 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024

... highlights how rapidly an organism can grow by cell division, for example. This series is also the basis of all computer science.

times, it is probably about 13 mm thick. Surely only a supercomputer could calculate how many more folds before you reach the moon!



Well, no, you and your class can do it, and you won't need so much as an abacus. Indeed, a calculator will only confuse matters, as it may not be able to cope with the large numbers that arise from repeated doubling.

There are two facts we need to know about the physical world before we can contemplate this calculation. The distance to the moon is around 400,000 km. If you measure a ream of photocopier paper, it will be about 50 mm thick and contain 500 sheets. So, you don't need a calculator to work out that a single sheet is about 0.1 mm thick.

The key to the calculation is noticing that the tenth number in the series above, 1,024, is so close to 1,000. Indeed, computer shops use it as the unit for memory. For example, a kilobyte is not 1,000 but 1,024 bytes and a megabyte is not 1,000,000 but 1,048,576. If the people who sell computers can use this approximation, then so can we.

Every ten folds, the paper increases in thickness by a factor of about 1,000. So, after the first ten folds it would be 100 mm thick. After another 10 folds, it would be 100,000 mm (or 100 m) thick. After the thirtieth fold it would be 100,000 m or 100 km thick. After the fortieth fold it would be about 100,000 km thick. That leaves us short of the moon by a factor of four, which only requires two more folds.

By this series, you can see how the folded paper is first twice, then four times, eight times, sixteen times etc thicker than it was. If you look at the piece of paper you folded six

So, as all fans of the Hitchhiker's Guide to the Galaxy will relish, the answer is 42 and it was easy, even without a calculator.